

What Is Claimed Is:

- 1 1. A method for using a computer system to solve a global
2 optimization problem specified by a function f and a set of equality constraints,
3 the method comprising:
4 receiving a representation of the function f and the set of equality
5 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) at the computer system, wherein f is a scalar
6 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
7 storing the representation in a memory within the computer system;
8 performing an interval equality constrained global optimization process to
9 compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
10 subject to the set of equality constraints;
11 wherein performing the interval equality constrained global optimization
12 process involves,
13 applying term consistency to a set of relations associated
14 with the interval equality constrained global optimization problem
15 over a subbox \mathbf{X} , and excluding any portion of the subbox \mathbf{X} that
16 violates any of these relations,
17 applying box consistency to the set of relations associated
18 with the interval equality constrained global optimization problem
19 over the subbox \mathbf{X} , and excluding any portion of the subbox \mathbf{X} that
20 violates any of the relations, and
21 performing an interval Newton step for the interval
22 equality constrained global optimization problem over the subbox
23 \mathbf{X} .

1 2. The method of claim 1, wherein applying term consistency to the
2 set of relations involves applying term consistency to the set of equality
3 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) over the subbox \mathbf{X} .

1 3. The method of claim 1, wherein applying box consistency to the
2 set of relations involves applying box consistency to the set of equality constraints
3 $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) over the subbox \mathbf{X} .

1 4. The method of claim 1,
2 wherein performing the interval equality constrained global optimization
3 process involves,
4 keeping track of a least upper bound f_bar of the function
5 $f(\mathbf{x})$, and
6 removing from consideration any subbox for which
7 $\inf(f(\mathbf{X})) > f_bar$;
8 wherein applying term consistency to the set of relations involves applying
9 term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 5. The method of claim 4, wherein applying box consistency to the
2 set of relations involves applying box consistency to the f_bar inequality
3 $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 6. The method of claim 1,
2 wherein performing the interval equality constrained global optimization process
3 involves preconditioning the set of equality constraints through multiplication by
4 an approximate inverse matrix \mathbf{B} to produce a set of preconditioned equality
5 constraints; and

6 wherein applying term consistency to the set of relations involves applying
7 term consistency to the set of preconditioned equality constraints over the subbox
8 **X**.

1 7. The method of claim 6, wherein applying box consistency to the
2 set of relations involves applying box consistency to the set of preconditioned
3 equality constraints over the subbox **X**.

1 8. The method of claim 1, wherein performing the interval Newton
2 step involves performing the interval Newton step on the John conditions.

1 9. The method of claim 1, wherein prior to performing the interval
2 Newton step on the John conditions, the method further comprises performing a
3 linearization test to determine whether to perform the Newton step on the John
4 conditions.

1 10. The method of claim 1, wherein performing the interval equality
2 constrained global optimization process involves:
3 evaluating a first termination condition;
4 wherein the first termination condition is TRUE if the width of the subbox
5 **X** is less than a pre-specified value, ε_X , and the width of $f(\mathbf{X})$ is less than a pre-
6 specified value, ε_F ; and
7 if the first termination condition is TRUE, terminating further splitting of
8 the subbox **X**.

1 11. The method of claim 1, wherein performing the interval Newton
2 step involves:

1 computing $J(\mathbf{x}, \mathbf{X})$, wherein $J(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f}
2 evaluated as a function of \mathbf{x} over the subbox \mathbf{X} ; and
3 determining if $J(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
4 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where
5 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}J(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
6 $J(\mathbf{x}, \mathbf{X})$.

1 12. A computer-readable storage medium storing instructions that
2 when executed by a computer cause the computer to perform a method for using a
3 computer system to solve a global optimization problem specified by a function f
4 and a set of equality constraints, the method comprising:
5 receiving a representation of the function f and the set of equality
6 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) at the computer system, wherein f is a scalar
7 function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
8 storing the representation in a memory within the computer system;
9 performing an interval equality constrained global optimization process to
10 compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$
11 subject to the set of equality constraints;
12 wherein performing the interval equality constrained global optimization
13 process involves,
14 applying term consistency to a set of relations associated
15 with the interval equality constrained global optimization problem
16 over a subbox \mathbf{X} , and excluding any portion of the subbox \mathbf{X} that
17 violates any of these relations,
18 applying box consistency to the set of relations associated
19 with the interval equality constrained global optimization problem

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20 over the subbox **X**, and excluding any portion of the subbox **X** that
21 violates any of the relations, and
22 performing an interval Newton step for the interval
23 equality constrained global optimization problem over the subbox
24 **X**.

1 13. The computer-readable storage medium of claim 12, wherein
2 applying term consistency to the set of relations involves applying term
3 consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) over the subbox
4 **X**.

1 14. The computer-readable storage medium of claim 12, wherein
2 applying box consistency to the set of relations involves applying box consistency
3 to the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) over the subbox **X**.

1 15. The computer-readable storage medium of claim 12,
2 wherein performing the interval equality constrained global optimization
3 process involves,
4 keeping track of a least upper bound f_bar of the function
5 $f(\mathbf{x})$, and
6 removing from consideration any subbox for which
7 $\inf(f(\mathbf{X})) > f_bar$;
8 wherein applying term consistency to the set of relations involves applying
9 term consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the subbox **X**.

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1 16. The computer-readable storage medium of claim 15, wherein
2 applying box consistency to the set of relations involves applying box consistency
3 to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 17. The computer-readable storage medium of claim 12,
2 wherein performing the interval equality constrained global optimization
3 process involves preconditioning the set of equality constraints through
4 multiplication by an approximate inverse matrix \mathbf{B} to produce a set of
5 preconditioned equality constraints; and
6 wherein applying term consistency to the set of relations involves applying
7 term consistency to the set of preconditioned equality constraints over the subbox
8 \mathbf{X} .

1 18. The computer-readable storage medium of claim 17, wherein
2 applying box consistency to the set of relations involves applying box consistency
3 to the set of preconditioned equality constraints over the subbox \mathbf{X} .

1 19. The computer-readable storage medium of claim 12, wherein
2 performing the interval Newton step involves performing the interval Newton step
3 on the John conditions.

1 20. The computer-readable storage medium of claim 12, wherein prior
2 to performing the interval Newton step on the John conditions, the method further
3 comprises performing a linearization test to determine whether to perform the
4 Newton step on the John conditions.

1 21. The computer-readable storage medium of claim 12, wherein
 2 performing the interval equality constrained global optimization process involves:
 3 evaluating a first termination condition;
 4 wherein the first termination condition is TRUE if the width of the subbox
 5 \mathbf{X} is less than a pre-specified value, ε_X , and the width of the $f(\mathbf{X})$ is less than a pre-
 6 specified value, ε_F ; and
 7 if the first termination condition is TRUE, terminating further splitting of
 8 the subbox \mathbf{X} .

1 22. The computer-readable storage medium of claim 12, wherein
 2 performing the interval Newton step involves:
 3 computing $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f}
 4 evaluated as a function of \mathbf{x} over the subbox \mathbf{X} ; and
 5 determining if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
 6 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where
 7 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
 8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.

1 23. An apparatus that solves a global optimization problem specified
 2 by a function f and a set of equality constraints, the apparatus comprising:
 3 a receiving mechanism that is configured to receive a representation of the
 4 function f and the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$), wherein f is a
 5 scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$;
 6 a memory for storing the representation;
 7 an interval global optimization mechanism that is configured to perform
 8 an interval equality constrained global optimization process to compute

9 guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$ subject to the
10 set of equality constraints;

11 a term consistency mechanism within the interval global optimization
12 mechanism that is configured to apply term consistency to a set of relations
13 associated with the interval equality constrained global optimization problem over
14 a subbox \mathbf{X} , and to exclude any portion of the subbox \mathbf{X} that violates the set of
15 relations;

16 a box consistency mechanism within the interval global optimization
17 mechanism that is configured to apply box consistency to the set of relations
18 associated with the interval equality constrained global optimization problem over
19 the subbox \mathbf{X} , and to exclude any portion of the subbox \mathbf{X} that violates the set of
20 relations; and

21 an interval Newton mechanism within the interval global optimization
22 mechanism that is configured to perform an interval Newton step for the interval
23 equality constrained global optimization problem over the subbox \mathbf{X} .

1 24. The apparatus of claim 23, wherein the term consistency
2 mechanism is configured to apply term consistency to the set of equality
3 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) over the subbox \mathbf{X} .

1 25. The apparatus of claim 23, wherein the box consistency
2 mechanism is configured to apply box consistency to the set of equality
3 constraints $q_i(\mathbf{x}) = 0$ ($i=1, \dots, r$) over the subbox \mathbf{X} .

1 26. The apparatus of claim 23,
2 wherein the interval global optimization mechanism is configured to,

3 keep track of a least upper bound f_bar of the function $f(\mathbf{x})$,
4 and to
5 remove from consideration any subbox for which
6 $\inf(f(\mathbf{X})) > f_bar$;
7 wherein the term consistency mechanism is configured to apply term
8 consistency to the f_bar inequality $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 27. The apparatus of claim 26, wherein the box consistency
2 mechanism is configured to apply box consistency to the f_bar inequality
3 $f(\mathbf{x}) \leq f_bar$ over the subbox \mathbf{X} .

1 28. The apparatus of claim 23,
2 wherein the interval global optimization mechanism is configured to
3 precondition the set of equality constraints through multiplication by an
4 approximate inverse matrix \mathbf{B} to produce a set of preconditioned equality
5 constraints; and
6 wherein the term consistency mechanism is configured to apply term
7 consistency to the set of preconditioned equality constraints over the subbox \mathbf{X} .

1 29. The apparatus of claim 28, wherein the box consistency
2 mechanism is configured to apply box consistency to the set of preconditioned
3 equality constraints over the subbox \mathbf{X} .

1 30. The apparatus of claim 23, wherein the interval Newton
2 mechanism is configured to perform the interval Newton step on the John
3 conditions.

1 31. The apparatus of claim 23, wherein prior to performing the interval
2 Newton step on the John conditions, the interval global optimization mechanism
3 is configured to perform a linearization test to determine whether to perform the
4 Newton step on the John conditions.

1 32. The apparatus of claim 23, wherein the interval global optimization
2 mechanism is configured to:
3 evaluate a first termination condition, wherein the first termination
4 condition is TRUE if the width of the subbox \mathbf{X} is less than a pre-specified value,
5 ε_X , and the width of $f(\mathbf{X})$ is less than a pre-specified value, ε_F ; and to
6 terminate further splitting of the subbox \mathbf{X} if the first termination
7 condition is TRUE.

1 33. The apparatus of claim 23, wherein the interval Newton
2 mechanism is configured to:
3 compute $\mathbf{J}(\mathbf{x}, \mathbf{X})$, wherein $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is the Jacobian of the function \mathbf{f} evaluated
4 as a function of \mathbf{x} over the subbox \mathbf{X} ; and to
5 determine if $\mathbf{J}(\mathbf{x}, \mathbf{X})$ is regular as a byproduct of solving for the subbox \mathbf{Y}
6 that contains values of \mathbf{y} that satisfy $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$, where
7 $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$, $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$, and \mathbf{B} is an approximate inverse of the center of
8 $\mathbf{J}(\mathbf{x}, \mathbf{X})$.